# Negative individuals

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- Introducing negative individuals: enriching the domain of individuals with a notion of polarity.
- Applying negative individuals to the semantics of numeral expressions.
    - The problem of 'zero'.
    - The problem of unattested negative entailments.
    - van Benthem's problem.
    - Cumulative readings.
- Generalizing the treatment of numerals: all quantificational determiners can be treated as predicates. Ramifications for:
    - Conservativity.
    - Quantificational scope.
- Tentative extension to *but*-exceptives.

# Individuals and their negative counterparts

- Consider the following sentence:

(1) Ashton but not Sam smokes.

- Bledin (2024): If we take the surface syntax at face value, we need a semantics for 'not' that may apply to an individual, and return an individual (building on Akiba 2009, Fine 2017).

- Bledin's conjecture: just as matter stands in relation to anti-matter, **Sam** stands in relation to **not Sam**.
- 'not Sam' is a bona fide negative individual.

(2) $[\![\text{not Sam}]\!] = \text{Sam}^-$

- **Sam**$^-$ is **Sam**'s 'negative counterpart' (no relation to Lewisian counterpart theory).
- To distinguish ordinary individuals, I'll write **Sam**$^+$.
- Individuals are thereby *polarized*.

- Collective conjunction takes two individuals $a, b$, and forms a group $a \sqcup b$ (Link 1983, Landman 1989a,b) ; this extends to *polarized* individuals too.

(3) $$\begin{aligned} [\![\text{Ashton and not Sam}]\!] &= [\![\text{Ashton}]\!] \sqcup [\![\text{not Sam}]\!] \\ &= \text{Ashton}^{+} \sqcup \text{Sam}^{-} \end{aligned}$$

- An immediate question: what does it mean for a *negative individual* to be true of a predicate?
- Easy, *not Sam* is true of *smokes* iff *Sam* herself is false of *smokes*.

(4) 'Polarized' function application:

a. $[\![\text{smokes}]\!]\,(\text{Ashton}^{+}) \iff$ Ashton smokes

b. $[\![\text{smokes}]\!]\,(\text{Sam}^{-}) \iff$ Sam doesn't smoke

- *Groups* of polarized individuals compose with predicates via an implicit universal quantifier $\Delta$ (analogous to Link's distributivity operator):
- $\Delta$ applies to a predicate defined for 'ordinary' positive atoms, and returns a predicate of groups of polarized individuals, which universally quantifies over atomic parts.

(5) [Ashton but not Sam] $\Delta$ smokes.

$$\Longleftrightarrow \Delta(\llbracket \text{smokes} \rrbracket)(\llbracket \text{Ashton but not Sam} \rrbracket)$$

$$\Longleftrightarrow [\lambda X\,.\,\forall x \leq_{At} X, \llbracket \text{smokes} \rrbracket\,(x)](\text{Ashton}^{+} \sqcup \text{Sam}^{-})$$

$$\Longleftrightarrow \text{Ashton smokes and Sam does not}$$

# Technicalities: polarizing the domain

- The domain of *polarized* individuals $D^{\pm}$ is built on a 'base' domain $D$, and then closed under group formation $\sqcup$.
- An important assumption: incoherent groups are filtered out (Akiba 2009).

(6) $D = \{ a, b, c, \dots \}$

(7) $$D^{\pm} = \left\{ \begin{array}{c} a^{+}, a^{-}, b^{+}, b^{-}, c^{+}, c^{-}, \dots \\ \cancel{a^{+} \sqcup a^{-}}, a^{+} \sqcup b^{+}, a^{+} \sqcup b^{-}, a^{+} \sqcup c^{+}, a^{+} \sqcup c^{-} \\ a^{-} \sqcup b^{+}, a^{-} \sqcup b^{-}, a^{-} \sqcup c^{+}, a^{-} \sqcup c^{-} \\ \dots \end{array} \right\}$$

- A group of polarized individuals is incoherent if, for any $x \in D$, it contains atomic parts $x^{+}$ and $x^{-}$.

- Since incoherent groups are filtered out, an interesting property of $D^{\pm}$ is that it has *multiple maximal elements* (with respect to parthood $\leq$).
  - Given a base domain $\{a, b, c\}$ the maxima are given below:

$$(8)\quad \left\{ \begin{array}{c} a^+ \sqcup b^+ \sqcup c^+ \\ a^+ \sqcup b^+ \sqcup c^-, a^+ \sqcup b^- \sqcup c^+, a^- \sqcup b^+ \sqcup c^+ \\ a^+ \sqcup b^- \sqcup c^-, a^- \sqcup b^+ \sqcup c^-, a^- \sqcup b^- \sqcup c^+ \\ a^- \sqcup b^- \sqcup c^- \end{array} \right\}$$

- In developing a semantics for numerals/other quantifiers, reference to such *maximal* elements will be a central component of the analysis.

- Negative individuals might be considered to be ontologically suspect. What exactly are they?
- A more conservative outlook: *a polarized individual is nothing more, and nothing less than an ordinary individual accompanied by a single bit of information* (Bledin 2024, Simon Charlow p.c.).

(9) $\text{Ashton}^{+} := \langle \text{Ashton}, 1 \rangle$

(10) $\text{Sam}^{-} := \langle \text{Sam}, 0 \rangle$

- Polarized individuals can be conceived of as *individual truth-value pairs*; everything I say today can be reconstructed in these terms.

- Following Bledin (2024), I've suggested an enrichment of the domain of individuals: $D^{\pm}$.
- $D^{\pm}$ contains both ordinary 'positive' individuals, negative individuals, as well as groups formed from these things.
- Bledin focuses on 'not' in conjunctions—see Bledin 2024 for further argumentation, focusing on negation and collective conjunction.
- In the following, I will exploit negative individuals *indirectly*, in order to develop a new decompositional semantics for numeral expressions.

# Numeral expressions

- There are two standard approaches to the semantics of numeral expressions in the literature.
  - The first treats numerals as Generalized Quantifiers (GQs) (Barwise & Cooper 1981); they count the intersection of the restrictor and the scope.
  - The second treats numerals as *cardinality predicates*; they count atomic parts of groups (Winter 2001).

(11) Three boys smoke.

a. $\#(B \cap S) \geq 3$ GQ theory

b. $\exists X, \text{boys}(X), \#(At^{+}(X)) = 3, \forall x \leq_{At} X, x$ smokes
*There's a group of boys with three atomic parts, each of whom smokes* predicative theory

# Two perspectives ii

- Both GQ-theoretic and the cardinality predicate approaches have distinct advantages and disadvantages.
- Problems for GQ-theory: fails to capture the intuition that numeral expressions introduce *groups*.
    - Can't account for plural predication, "three boys met"…
    - …or cumulative readings (to be discussed later).
- The cardinality predicate approach struggles with:
    - Unattested existential entailments for 'less than *n*'.
    - Upper-bounded numerals like 'at most *n*' ('van Benthem's problem').
- My positive proposal can be seen as a hybrid—negative individuals allow for a version of the cardinality predicate theory which maintains the advantages of the GQ-theoretic approach.

- Given a group of polarized individuals, we can extract (a) the *positive atoms*, (b) the *negative atoms*.

(12) a. $At^{+}(a^{+} \sqcup b^{-} \sqcup c^{+}) = \{a, c\}$

b. $At^{-}(a^{+} \sqcup b^{-} \sqcup c^{+}) = \{b\}$

c. $At^{\pm}(X) := A^{+}(X) \cup A^{-}(X)$

- Proposal for numerals: numerals count positive atoms; they have a basic *at least* semantics.

(13) $[\![\text{two}]\!](X) \iff \#(At^{+}(X)) \geq 2$

- NPs (crucially) range over *maximal* groups of polarized individuals.

(14) $[\![\text{boy}]\!] = Max_{\leq}\{X \in D^{\pm} \mid \forall x \in At^{\pm}(X), x \text{ is a boy}\}$

- Assuming the boys $b_1, b_2, b_3$, since incoherent pluralities are filtered, there are multiple maxima:

(15) $$[\![\text{boy}]\!] = \left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^- \sqcup b_3^-, b_1^- \sqcup b_2^+ \sqcup b_3^-, b_1^- \sqcup b_2^- \sqcup b_3^+, \\ b_1^- \sqcup b_2^- \sqcup b_3^- \end{array} \right\}$$

- A useful metaphor: each element says, for each boy, where or not he is true of some yet-to-be-specified predicate $P$

- Both NPs and numerals are *predicates of groups of polarized individuals*, and therefore may compose via intersective modification (e.g., via Heim & Kratzer's rule of predicate modification).

(16) $$[\![\text{two}]\!] \cap [\![\text{boy}]\!] = \{X \in D^{\pm} \mid [\![\text{boy}]\!](X), \#(At^{+}(X)) \geq 2\}$$
$$= \left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^- \sqcup b_3^+}, \\ \cancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array} \right\}$$

- *Two* filters out any group with fewer than two positive atoms.

(17) Two boys sneezed.

- We still need to (somehow) compose a *set of groups* with an ordinary predicate of individuals. We adopt a standard technique from the literature on numerals:
    - the NP 'two boys' combines with an implicit existential quantifier $\exists$ (Partee 1986, Winter 2001).
    - The resulting quantifier composes distributively, via $\Delta$.

(18) [$\exists$ two boys] [$\Delta$ sneezed].

# Composing numerals iii

1 iff at least two boys sneezed

$$\lambda P\,.\,\exists X \in \left\{\begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, \\ b_1^+ \sqcup b_2^- \sqcup b_3^+, \\ b_1^- \sqcup b_2^+ \sqcup b_3^+ \end{array}\right\}, P(X) \qquad \lambda X\,.\,\forall x \leq_{At} X, [\![\text{sneezed}]\!]\,(x)$$

$\exists$ $\quad$ $[\![\text{two}]\!] \cap [\![\text{boys}]\!]$ $\qquad$ $\Delta$ $\quad$ sneezed

two $\quad$ boys

- Predicates of groups of *polarized* individuals are expressive enough to account not just for bare numerals, but also for modified numerals.
    - $[\![\text{exactly two}]\!]\,(X) \iff \#(At^{+}(X)) = 2$
    - $[\![\text{less than two}]\!]\,(X) \iff \#(At^{+}(X)) < 2$

(19) $$[\![\text{exactly two}]\!] \cap [\![\text{boys}]\!] = \left\{ \begin{array}{c} \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^+}, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^- \sqcup b_3^+}, \\ \cancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array} \right\}$$

(20) $$[\![\text{less than two}]\!] \cap [\![\text{boys}]\!] = \left\{ \begin{array}{c} \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^+}, \\ \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^+}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^+}, \\ b_1^+ \sqcup b_2^- \sqcup b_3^-, b_1^- \sqcup b_2^+ \sqcup b_3^-, b_1^- \sqcup b_2^- \sqcup b_3^+, \\ b_1^- \sqcup b_2^- \sqcup b_3^- \end{array} \right\}$$

- While resembling the cardinality predicate account, the polarized account shares a number of advantages with GQ theory:
  - The numeral *zero* is expressible as a cardinality predicate.
  - Unattested existential entailments 'less than *n*' are avoided.
  - Issues for upper-bounded numerals like 'at most/exactly *n*' don't arise.
- The polarized theory will furthermore naturally account for cumulative readings, since numerals range over groups.
- These points are covered in more detail in the following section.

# ‘Zero’ as a cardinality predicate

- Numerals as cardinality predicates classically: no plurality has zero atomic parts.
  - ‘zero’ must be treated as a special case, e.g., as a generalized quantifier.
  - Alternatively a special ‘empty’ plurality may be posited (Bylinina & Nouwen 2018, Buccola & Spector 2016).
- In a polarized setting, this isn’t necessary; ‘zero NP’ picks out the unique, maximal, wholly-negative group.

(21) $[\![\text{zero}]\!]\,(X) \iff \#(At^{+}(X)) = 0$

(22) Zero boys sneezed.

$\iff \exists X \in [\![\text{zero boys}]\!]\,, \Delta([\![\text{sneezed}]\!])(X)$

$\iff \exists X \in \{\, b_1^- \sqcup b_2^- \sqcup b_3^- \,\}, \Delta([\![\text{sneezed}]\!])(X)$

$\iff \forall x \leq_{At} b_1^- \sqcup b_2^- \sqcup b_3^-, [\![\text{sneezed}]\!]\,(x)$

$\iff$ *none of the boys sneezed.*

- The classical account: since no plurality has zero atomic parts, ‘less than two boys sneezed’ entails that at least one boy sneezed (Buccola & Spector 2016).
- The polarized account: the presence of the wholly-negative group avoids an unattested existential entailment.

(23) $[\![\text{less than two}]\!] \cap [\![\text{boys}]\!] =$

$$\left\{\begin{array}{c} \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^+}, \\ \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^+}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^+}, \\ b_1^+ \sqcup b_2^- \sqcup b_3^-, b_1^- \sqcup b_2^+ \sqcup b_3^-, b_1^- \sqcup b_2^- \sqcup b_3^+, \\ b_1^- \sqcup b_2^- \sqcup b_3^- \end{array}\right\}$$

- On the classical view of numerals as cardinality predicates, existential quantification renders upper-bounds inert.
  - The following statements are equivalent:

(24) $\exists X$, $X$ is a group of boys and $\#X = 3$, and $X$ each sneezed.

(25) $\exists X$, $X$ is a group of boys and $\#X \geq 3$, and $X$ each sneezed.

- This precludes a treatment of upper-bounded numerals such as 'exactly $n$' as simple cardinality predicates.

- van Benthem's problem now doesn't arise, since every group encodes *maximal* information.

(26) $[\![\text{exactly two}]\!]\,(X) \iff \#(At^{+}(X)) = 2$

(27) $[\![\text{exactly two}]\!] \cap [\![\text{boys}]\!] =$

$$\left\{\begin{array}{c} \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^+}, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^- \sqcup b_3^+}, \\ \cancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array}\right\}$$

- The upper-bound is encoded already at the level of the DP!

- NP-denotations provide complete answers to the question "which of the NP individuals did $P$" (for some yet-to-be-specified $P$).
- Numerals-as-modifiers *restrict* the set of possible answers, by applying a cardinality constraint to the positive atoms.

(28) $[\![\text{two}]\!](X) \iff \#(At^{+}(X)) \geq 2$

(29) $[\![\text{exactly two}]\!](X) \iff \#(At^{+}(X)) = 2$

(30) $[\![\text{less than two}]\!](X) \iff \#(At^{+}(X)) < 2$

(31) $[\![\text{zero}]\!](X) \iff \#(At^{+}(X)) = 0$

- In the following, I'll apply this approach to one of the hardest nuts to crack in numeral semantics: the interaction of upper-boundedness with cumulative readings .

- Plural expressions give rise to *cumulative readings*.

(32) Enrico and Filipe have read $b_1$, $b_2$ and $b_3$.
⇒ *Enrico and Filipe each read one of* $b_{1\ldots3}$.
⇒ *Each of* $b_{1\ldots3}$ *was read by either Enrico or Filipe.*

- N.b. sentences involving multiple plural arguments also have distributive readings; one way of biasing the cumulative reading is to use the modifier '…between them'.

(33) Enrico, Filipe but not Stan have read $b_1$, $b_2$, $b_3$ but not $b_4$.
⇒ *Enrico and Filipe each read one of* $b_{1\ldots3}$.
⇒ *Each of* $b_{1\ldots3}$ *have been read by either Enrico or Filipe.*
⇒ *Stan hasn't read any of* $b_{1\ldots4}$
⇒ $b_4$ *hasn't been read by Enrico, Filipe, or Stan.*

- Intuition: negative individuals interact with cumulativity in a characteristic fashion; they lead to complete non-participation inferences.

- Cumulativity standardly encoded by an operator .** (Beck & Sauerland 2000).
    - Conjecture: cumulativity is *polarity sensitive*.

(34) $$^{**}R(X,Y) \iff \forall x \in At^{+}(X), \exists y \in At^{+}(Y), R(x,y) \\ \wedge \forall y \in At^{+}(Y), \exists x \in At^{+}(X), R(x,y) \\ \wedge \neg\exists x \in At^{-}(X), y \in At^{\pm}(Y), R(x,y) \\ \wedge \neg\exists y \in At^{-}(Y), x \in At^{\pm}(X), R(x,y)$$

- The first two lines say that $At^{+}(X)$ and $At^{+}(Y)$ stand cumulatively in the $R$ relation.
- The final two lines encode non-participation of negative individuals.

- The problem of cumulative readings with modified numerals:

(35) Exactly two students read exactly three books.

- Truth-conditions, informally:
  - Some students read some books.
  - Tallying up the book-reading students: there are no more, and no less than two.
  - Tallying up the books-read-by-students: there are no more, and no less than three.
- The problem: the upper-bounds imposed by *exactly n* apply globally; they do not scopally interact (Landman 2000).

# Cumulative readings and modified numerals ii

- Why is this a problem?
    - In a more standard setting, upper-bounds are imposed via a *maximality* condition.
    - Maximality conditions take scope; their interaction with cumulativity seems to be at odds with compositionality (Krifka 1999).
- See especially (Brasoveanu 2013) on why the standard picture fails to derive attested cumulative readings.
- Existing appoaches to this problem exploit powerful mechanisms for side-stepping scopal interactions, such as post-suppositions (Brasoveanu 2013, Charlow 2021, Haslinger & Schmitt 2020).

- The current account immediately predicts this interaction; modified numerals are merely existential quantifiers over groups of polarized individuals.
- Existential quantifiers scopally-commute.

(36) Exactly two students read exactly two books.
$\exists X \in [\![\text{ex. 2 students}]\!], \exists Y \in [\![\text{ex. 2 books}]\!],^{**} [\![\text{read}]\!](X, Y)$

(37) $$\exists X \in \left\{ \begin{array}{c} s_1^+ \sqcup s_2^+ \sqcup s_3^-, \\ s_1^+ \sqcup s_2^- \sqcup s_3^+ \\ s_1^- \sqcup s_2^+ \sqcup s_3^+ \end{array} \right\}, \exists Y \in \left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^-, \\ b_1^+ \sqcup b_2^- \sqcup b_3^+ \\ b_1^- \sqcup b_2^+ \sqcup b_3^+ \end{array} \right\},^{**} [\![\text{read}]\!](X, Y)$$

- The predicted truth conditions are disjunctive; given students $s_1, s_2, s_3$ and books $b_1, b_2, b_3$, either:
  - $s_1, s_2$ read $b_1, b_2$ between them; $b_3$ didn't read any book, and $s_3$ wasn't read by anyone, or…
  - … $s_1, s_3$ read $b_1, b_2$ between them; $s_2$ didn't read any book, and $b_3$ wasn't read by anyone, or…
  - … $s_2, s_3$ read $b_1, b_2$ between them; $s_1$ didn't read any book, and $b_3$ wasn't read by anyone, etc.
- In general: some students read some books.
  - Tallying up the book-reading students, there are exactly two.
  - Tallying up the books-read-by-students, there are exactly two.
- Since upper-bounds are *encoded in the groups that DPs range over*, no scopal interactions are expected.

- The following example also has a (doubly) *distributive* reading, compatible if four (or more) books having been read.
    - Informally: *exactly two boys are s.t., they each read exactly two books; the other boys read either more than two books, or less than two.*

(38) Exactly two students read exactly two books.

- This follows from allowing modified numerals to independently take distributive scope, as on any theory.

(39) $\exists X \in [\![\text{ex. 2 students}]\!]\ (\Delta(\lambda x\,.\,\exists Y \in [\![\text{ex. 2 books}]\!]\ (\Delta(\lambda y\,.\,R(x,y))(Y))))(X)$

- A novel prediction: *zero* is an ordinary numeral. It should support cumulative readings.

(40) Context: *a college professor has asked their students to submit questions in advance of their weekly seminar, and each student is allowed to submit more than one question. The professor asks their teaching assistant how many questions were submitted (fearing the worst). Their teaching assistant responds:*
Unfortunately, zero students submitted zero questions this week.

- $^{**}S(s_1^- \sqcup s_2^- \sqcup s_3^- \sqcup \ldots, q_1^- \sqcup q_2^- \sqcup q_3^- \sqcup \ldots)$

- Some naturally occurring examples:

(41) "Dear Reader, you need to unplug yourself because nobody outside Reddit cares or even knows about what arguments are going on there. I've met **zero** people in **zero** places in meat space that have heard of Rippetoe or Mehdi, […] or any other God we cared to hold up…" (my emphasis)
`https://web.archive.org/web/20250212095935/https://purplespengler.blogspot.com/`

(42) "At the time btw **zero** people gave **zero** fucks about gain-of-function research. Trump cleared that in US between 2017 and 2020." (my emphasis)
`https://web.archive.org/web/20250212100005/https://pokerfraudalert.com/forum/showthread.php?19983-So-coronavirus-is-definitely-going-to-kill-a-few-of-us%252Fpage638`

- We've seen that incorporating polarity in the individual domain allows us to 'upgrade' the semantics of numerals as cardinality predicates.
- Groups of polarized individuals effectively encode maximal information at the level of the DP, allowing for straightforward accounts of data that vexes more standard approaches.
- Why stop at numeral expressions? In the following, I explore the idea that *all determiners* can be characterized as predicates.

# Determiners as predicates

- Recall, NPs denote *maximal* groups of polarized individuals.
  - Filtering out incoherent groups results in multiple maxima.

$$(43) \quad [\![\text{boy}]\!] = \left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^- \sqcup b_3^-, b_1^- \sqcup b_2^+ \sqcup b_3^-, b_1^- \sqcup b_2^- \sqcup b_3^+, \\ b_1^- \sqcup b_2^- \sqcup b_3^- \end{array} \right\}$$

- Each group corresponds to a *complete answer* to the question: 'which of $b_1, b_2, b_2$ did $P$?' (for some yet-to-be-specified predicate $P$).

- Perhaps unsurprisingly, this perspective immediately extends to quantificational determiners more generally.

(44) $[\![\text{some}]\!] = \{X \in D^{\pm} \mid At^{+}(X) \neq \varnothing\}$

(45) $[\![\text{every}]\!] = \{X \in D^{\pm} \mid At^{-}(X) = \varnothing\}$

- ‘some’ is true of any group with at least some positive atoms; ‘all’ is true of any group with no negative atoms.
  - Much like numerals determiners may compose with NPs as intersective modifiers (e.g., via the rule of *Predicate Modification* (Heim & Kratzer 1998)).

(46) $[\![\text{some}]\!] \cap [\![\text{boy}]\!] =$

$$\left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^- \sqcup b_3^-, b_1^- \sqcup b_2^+ \sqcup b_3^-, b_1^- \sqcup b_2^- \sqcup b_3^+, \\ \xcancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array} \right\}$$

(47) Some boy sneezed.

$\exists X \in [\![\text{some}]\!] \cap [\![\text{boy}]\!], \Delta([\![\text{sneezed}]\!])(X)$

$\iff \exists x[x \text{ is a boy and } x \text{ sneezed}]$

(48) $[\![\text{every}]\!] \cap [\![\text{boy}]\!] =$

$$\left\{\begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ \cancel{b_1^+ \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^+}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^+}, \\ \cancel{b_1^+ \sqcup b_2^- \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^+ \sqcup b_3^-}, \cancel{b_1^- \sqcup b_2^- \sqcup b_3^+}, \\ \cancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array}\right\}$$

(49) Every boy sneezed.

$\exists X \in [\![\text{every}]\!] \cap [\![\text{boy}]\!], \mathit{\Delta}([\![\text{sneezed}]\!])(X)$

$\iff \mathit{\Delta}([\![\text{sneezed}]\!])(b_1^+ \sqcup b_2^+ \sqcup b_3^+)$

$\iff \forall x[x \text{ is a boy} \rightarrow x \text{ sneezed}]$

- N.b., universal quantification *requires* reference to negative atoms.

- ‘no’ and ‘not every’ are straightforward definable via predicate negation of ‘some’ and ‘every’ respectively.

(50) $[\![\text{no}]\!] = \{X \in D^{\pm} \mid At^{+}(X) = \varnothing\}$

(51) $[\![\text{not every}]\!] = \{X \in D^{\pm} \mid At^{-}(X) \neq \varnothing\}$

- I’ll leave it as an exercise to verify that these entries result in the correct truth conditions.
  - Intersecting ‘no’ with an NP denotation results in a singleton set: the maximal, wholly-negative group.
  - Intersecting ‘not every’ with an NP denotation results in a non-singleton set.

- The standard ‘textbook’ approach to the semantics of determiners is provided by *Generalized Quantifier theory* (Barwise & Cooper 1981, Westerståhl 2024).
  - In GQ theory, determiner meanings are modeled as relations between sets or (equivalently) as higher-order functions (Montague 1973, Heim & Kratzer 1998).

(52) *Det* $\overbrace{\text{boy}}^{A}$ $\overbrace{\text{sneezed}}^{B}$.

(53) $\textbf{some}(A, B) \iff A \cap B \neq \varnothing$

(54) $\textbf{every}(A, B) \iff A \subseteq B$

(55) $\textbf{no}(A, B) \iff A \cap B = \varnothing$

- Some determiners are famously not first-order definable, and require the expressivity of GQ theory, e.g., proportional 'most'.
- Question: Is the expressivity of the predicative theory limited to first-order definable determiners?

(56) Most boys sneezed.

- In GQ theory:

(57) $\textbf{most}(A, B) \iff \#(A \cap B) > \#(A - B)$
Informally: *the As that B out-number the As that do not B.*

- In the predicative theory, the entry for 'most' is even simpler:

(58) $[\![\text{most}]\!] = \{\, X \mid \#(At^{+}(X)) > \#(At^{-}(X)) \,\}$

- Given an NP *A*, 'most' restricts the range of possible true answers to a question of the form 'which of the *A*s did *B*?'
  - It insists that any true answer entails that the *A*s that did *B* outweigh the *A*s that didn't do *B*.

(59) $[\![\text{most}]\!] \cap [\![\text{boy}]\!] =$

$$\left\{ \begin{array}{c} b_1^+ \sqcup b_2^+ \sqcup b_3^+, \\ b_1^+ \sqcup b_2^+ \sqcup b_3^-, b_1^+ \sqcup b_2^- \sqcup b_3^+, b_1^- \sqcup b_2^+ \sqcup b_3^+, \\ \xcancel{b_1^+ \sqcup b_2^- \sqcup b_3^-}, \xcancel{b_1^- \sqcup b_2^+ \sqcup b_3^-}, \xcancel{b_1^- \sqcup b_2^- \sqcup b_3^+}, \\ \xcancel{b_1^- \sqcup b_2^- \sqcup b_3^-} \end{array} \right\}$$

- In fact, we can state a mapping from GQ-theoretic determiners to predicates.
    - Let *Det* be a relation between sets.

(60) $Det_{Pred} := \{X \in D^{\pm} \mid Det(At^{\pm}(X), At^{+}(X))\}$

- (This mapping is truth-preserving only for *conservative* GQ-theoretic determiners; more on this in a bit).

- $\textbf{most}_{Pred} := \{ X \in D^{\pm} \mid \textbf{Most}(At^{\pm}(X), At^{+}(X)) \}$
  - $= \{ X \in D^{\pm} \mid \#(At^{\pm}(X) \cap At^{+}(X)) > \#(At^{\pm}(X) - At^{+}(X) \}$
  - $= \{ X \in D\pm \mid \#(At^{+}(X)) > \#(At^{-}(X)) \}$
- $\textbf{some}_{Pred} := \{ X \in D^{\pm} \mid \textbf{some}(At^{\pm}(X), At^{+}(X)) \}$
  - $= \{ X \in D^{\pm} \mid A^{\pm}(X) \cap A^{+}(X) \neq \varnothing \}$
  - $= \{ X \in D\pm \mid At^{+}(X) \neq \varnothing \}$
- $\textbf{every}_{Pred} := \{ X \in D^{\pm} \mid \textbf{every}(At^{\pm}(X), At^{+}(X)) \}$
  - $= \{ X \in D^{\pm} \mid A^{\pm}(X) \subseteq A^{+}(X) \}$
  - $= \{ X \in D\pm \mid At^{-}(X) = \varnothing \}$

- A natural question that arises at this point: how expressive is the predicative theory, compared to GQ theory?
- A famous linguistic universal: determiners in natural language express conservative GQ-theoretic determiners (Keenan & Stavi 1986).
- A GQ-theoretic determiner *Det* is conservative iff the following equivalence holds:

(61) $Det(A, B) \iff Det(A, A \cap B)$

- Informally, the non-$A$ $B$s are never be relevant for determining the truth of a quantificational statement '$Det(A, B)$', if *Det* is conservative.

- In GQ theory, it's straightforward to define non-conservative determiners.
  - A well-known example: the cardinality determiner $I$.

(62) $I(A, B) \iff \#A = \#B$

- Intuitively, this violates conservativity, since the entirety of $B$ is relevant to determining the truth of the statement, not just $A \cap B$.
- It's easiest to appreciate this with a simple illustration:
  - Let $A$ and $B$ be disjoint singleton sets $\{ a \}$ and $\{ b \}$, so $A \cap B = \varnothing$, $\#A = \#B = 1$.
  - $I(A, B)$ is true in this scenario, but $I(A, A \cap B)$ is false.

- The very fact that non-conservative determiners are stateable in GQ-theory leads to the question: why are they never lexicalized?
    - A tentative suggestion: non-conservative determiners are never lexicalized, because determiners aren't GQ-theoretic relations, but rather predicates of groups of polarized individuals.
    - Non-conservative determiners simply aren't stateable in the predicative theory.
- One way appreciate this: apply the recipe for mapping GQ-theoretic dets to preds to $I$.

- Assume: $A = \{a\}; B = \{b\}$, so $I(A, B)$ is true.

(63) $I_{Pred} := \{X \in D^{\pm} \mid I(At^{\pm}(X), At^{+}(X))\}$

(64) $I_{Pred} := \{X \in D^{\pm} \mid \#At^{\pm}(X) = \#At^{+}(X)\}$

(65) $I_{Pred} \cap A = \{a^{+}\}$

(66) $B(a^{+}) = 0$

- Informally, $I_Pred$ demands that the cardinality of $A$ is the same as the cardinality of $A \cap B$!
  - The mapping is not truth-preserving.
  - Fact: the mapping is *only* truth-preserving for conservative GQ-theoretic determiners.

- The predicative theory has the resources to express *all (and only) the conservative GQ-theoretic determiners*.
- Since determiners-as-predicates compose with $A$ (the NP restrictor) as intersective modifiers, they have the capacity to:
  - place restrictions on $A$, i.e., $At^{\pm}(X)$.
  - place restrictions on $A \cap B$, i.e., $At^{+}(X)$.
  - place restrictions on $A - B$, i.e., $At^{-}(X)$.
- This is because elements in the denotation of the NP restrictor correspond to complete answers to the question "which of the $A$s did $B$?".
- Determiners may only restrict this set; $B$-elements that are not also $A$s simply cannot be affected.

- To complete the picture, we can also map back from predicates to (conservative) GQ-theoretic determiners.
  - Given a predicate $Det_{Pred}$ the corresponding GQ-theoretic determiner is defined as follows.

(67) $Det_{GQ}(A, B)$

$$:= \exists X \in Det_{Pred} \cap Max_{\leq}\{X \in D^{\pm} \mid \forall x \in At^{\pm}(X), x \in A\}, \Delta(B)(X)$$

- The headline here: the predicative theory allows us to characterize a *subset* of the determiner meanings expressible in GQ-theory.
- Namely, only those corresponding to the *conservative* GQ-theoretic determiners.

- The resulting picture for the syntax-semantics interface ends up looking a little bit different.
    - DPs have two scope-taking components: *existential scope* and *distributive scope*.
    - Essentially this generalizes machinery from the literature on numeral semantics to all determiners.

(68) Yasu likes some student.

$\exists$ [some student] $\lambda X\ t_X\ \Delta\ [\lambda x$ Yasu likes $t_x]$

$\Rightarrow \exists X \in [\![\text{some student}]\!], \Delta(\lambda x\,.\,\text{Yasu likes } x)(X)$

- Scopal ambiguities arise because, even though existential quantifiers commute, the distributive quantifier $\Delta$ does not.
  - To illustrate, I'll go through how to derive an inverse scope reading.

(69) Some boy danced with every girl.
$\exists$ [every girl] $\lambda G\ t_G\ \Delta\ [\lambda g\,.\ \exists$ [some boy] $\lambda B\ t_B\ \Delta\ [\lambda b\ t_b$ danced with $t_g$]]
$\Rightarrow \exists G \in [\![\text{every girl}]\!], \Delta(\lambda g\,.\,\exists B \in [\![\text{some boy}]\!], \Delta(\lambda b\,.\,D(b,g))(B))(G)$
$\Rightarrow \exists G \in [\![\text{every girl}]\!], \Delta(\lambda g\,.\,\exists b, b$ is a boy and $b$ danced with $g)(G)$
$\Rightarrow \forall g, g$ is a girl, $\exists b, b$ is a boy and $b$ danced with g

- As discussed in the literature on numeral semantics, the existential and distributive scope of a numeral may be *split*.
    - The existential component takes exceptional scope, whereas the distributive component is strictly clause-bounded (Ruys 1992).

(70) If three relatives of mine die, I'll inherit a fortune.

- if..then > $\exists$ > $\Delta$.
    - *If any three relatives of mine all die, I'll inherit a fortune.*
- $\exists$ > if..then > $\Delta$
    - *if a particular three relatives of mine all die, I'll inherit a fortune.*

- If we generalize this machinery to *all* determiners, don't we (incorrectly) predict that, e.g., 'every', and 'no' give rise to exceptional scope readings?

(71) If every relative of mine dies, I'll inherit a fortune.

a. ✓if..then > $\forall$

b. ✗$\forall$ > if..then

(72) If no relative of mine dies, I'll be penniless.

a. ✓if..then > $\neg\exists$

b. ✗$\neg\exists$ > if..then

- Surprisingly, no! Since 'every NP' and 'no NP' denote *singleton sets* on the predicative theory, exceptional existential scope is *vacuous* (assuming the NP restrictor is interpreted transparently!).
- Assume relatives of mine $r_1, r_2, r_2$.

(73) If every relative of mine dies, I'll inherit a fortune.
$\exists X \in \{ r_1^+ \sqcup r_2^+ \sqcup r_3^+ \}$ if $\Delta(\llbracket \text{dies} \rrbracket)(X)$, I'll inherit a fortune.
$\iff$ if $\Delta(\llbracket \text{dies} \rrbracket)(r_1^+ \sqcup r_2^+ \sqcup r_3^+)$, I'll inherit a fortune.

(74) If no relative of mine dies, I'll be penniless.
$\exists X \in \{ r_1^- \sqcup r_2^- \sqcup r_3^- \}$ if $\Delta(\llbracket \text{dies} \rrbracket)(X)$, I'll be penniless.
$\iff$ if $\Delta(\llbracket \text{dies} \rrbracket)(r_1^- \sqcup r_2^- \sqcup r_3^-)$, I'll be penniless.

- See Schwarzschild (2002) on the vacuity of existential quantification over a singleton set.

- Any determiner that results in a non-singleton set is expected to result in detectable exceptional scope readings. This has already been observed for indefinites and bare numerals (Ruys 1992).
- I contend that modified numerals allow for exceptional scope readings too (contra Cresti 1995, Reinhart 1997, Ruys & Spector 2017).

(75) Context: *James is playing a variant of roulette involving two balls. The rules are simple: James can bet on any two numbers, and if the balls both land on the numbers James picked, he doubles his bet. In any other scenario, James loses his bet. Unbeknownst to James, the croupier has rigged the wheel: the two balls are guaranteed to land on predetermined numbers.*
If James bets on exactly two of these numbers, He'll double his bet.
(I just don't know which two)

- A prediction that I'm much less sure about: since proportional determiners result in non-singleton sets, they should give rise to exceptional scope readings too.

(76) Context: *James is playing a variant of roulette involving involving a single ball. He can bet on any amount of numbers, but the more numbers he bets on, the smaller his potential winnings. Unbeknownst to James, the croupier has rigged the wheel: the wheel includes numbers 1-20, and the ball is guaranteed to land on 1-15.*
?If James bets on most of these numbers, he'll at least win something.
(I just don't know which ones exactly)

# Another loose end: semantic singularity/plurality

- The predicative theory as it stands has nothing to say about, e.g., *some boys* vs. *some boy*.
- Relatedly, it's unclear how to generalize this account to *collective predication*.

(77) *Some boy gathered.

(78) Some boys gathered.

- Note that the predicative theory is no worse off than GQ-theory in this regard.

- One possibility: following (Bledin 2024), assume that the *base domain* is inherently plural.

(79) $^{*}D = \{\, a, b, c, a \oplus b, a \oplus c, b \oplus c, a \oplus b \oplus c, \ldots \}$

- Polarizing a plural base domain results in a rich, multi-layered structure:

(80)
$$^{*}D^{\pm} = \left\{ \begin{array}{c} a^{+}, a^{-}, b^{+}, b^{-}, c^{+}, c^{-} \\ (a \oplus b)^{+}, (a \oplus b)^{-}, \ldots \\ a^{+} \sqcup (a \oplus b)^{+}, a^{+} \sqcup (a \oplus b)^{-}, \ldots \\ a^{-} \sqcup (a \oplus b)^{+}, a^{-} \sqcup (a \oplus b)^{-} \ldots \end{array} \right\}$$

- Note that, e.g., $a^{-} \sqcup (a \oplus b)^{+}$ does not count as incoherent.

- Plural NPs denote *maximal* groups of polarized *pluralities*.
    - Let the boys be $a, b, c$ (I omit singletons here for exposition).

(81) $$[\![\text{boys}]\!] = \left\{ \begin{array}{c} (a \oplus b)^{+} \sqcup (a \oplus c)^{-} \sqcup (b \oplus c)^{-} \sqcup (a \oplus b \oplus c)^{-} \\ (a \oplus b)^{-} \sqcup (a \oplus c)^{+} \sqcup (b \oplus c)^{-} \sqcup (a \oplus b \oplus c)^{-} \\ (a \oplus b)^{-} \sqcup (a \oplus c)^{-} \sqcup (b \oplus c)^{+} \sqcup (a \oplus b \oplus c)^{-} \\ \ldots \end{array} \right\}$$

- Collective predicates are true of non-atomic pluralities, such as $a \oplus b$.

(82) Some boys gathered.
$\exists X \in [\![\text{boys}]\!], At^{+}(X) \neq \varnothing, \Delta([\![\text{gathered}]\!])(X)$
$\iff$ *there's a plurality of boys X, s.t., X gathered.*

- Singular 'some boy' is incompatible with a collective predicate such as 'gathered', because it denotes a set of groups of polarized *atoms*.
    - To address a possible point of confusion: since the system is multi-layered, $At^{+}(X)$ may return a set of non-atomic individuals.

# Extension: exceptives

- Note that this section is based on (very tentative) work in progress with Justin Bledin (Johns Hopkins).

- *But* exceptives modify quantifiers, and have the following properties.
    - *Otherness* entailment.
    - *Negative* entailment.
    - Restricted to universal and negative quantifiers.

(83) Every boy but Harry sneezed.

   a. ⇒ *the boys other than Harry sneezed.* (otherness entailment)

   b. ⇒ *Harry didn't sneeze.* (negative entailment)

(84) No boy but Harry sneezed.

   a. ⇒ *the boys other than Harry didn't sneeze* (otherness entailment)

   b. ⇒ *Harry did sneeze.* (negative entailment)

(85) *A boy/exactly one boy/two boys/most boys but Harry sneezed.

- A classic analysis of *but*-exceptives due to von Fintel 1993:
    - *but*-exceptives subtract individuals from the restrictor set (*otherness* entailment).
    - *but*-exceptives come with a requirement that the subtracted individual is the *least* exception set relative to the associated quantification (details omitted here) (*negative* entailment).
    - Successors refine this in various ways (Gajewski 2008, Hirsch 2016) but maintain this basic division of labor.

(86) Every boy but $b_3$ sneezed.

a. $b_1$, $b_2$ sneezed (otherness via domain subtraction).

b. $\forall S$, every (boy $-S$) sneezed $\rightarrow b_3 \in S$

$\Rightarrow b_3$ didn't sneeze (negative entailment via leastness)

- *but*-exceptives serve to flip the polarity of the excepted-individual.
  - Let the boys be $b_1, b_2, b_3$.

(87) a. $[\![\text{every boy}]\!] = \{\, b_1^+ \sqcup b_2^+ \sqcup b_3^+ \,\}$

b. $[\![\text{every boy but } b_3]\!] = \{\, b_1^+ \sqcup b_2^+ \sqcup b_3^- \,\}$

(88) a. $[\![\text{no boy}]\!] = \{\, b_1^- \sqcup b_2^- \sqcup b_3^- \,\}$

b. $[\![\text{no boy but } b_3]\!] = \{\, b_1^- \sqcup b_2^- \sqcup b_3^+ \,\}$

- Accounts for *otherness* and *negative* entailments directly.

## Motivation: cumulative readings

- Exceptives can co-occur with cumulative readings (crucially, *all* NP licenses cumulative readings).

(89) All the students but Stan read all the books but Syntactic Structures.

- Entails:
    - *The boys other than Stan read the books other than Syntactic Structures between them* (otherness + cumulativity).
    - *Stan didn't read any of the books.* (negative entailment)
    - *None of the boys read syntactic structures.* (negative entailment)
- No obvious way for the standard account to account for this reading, since *leastness* conditions interact scopally (logic much the same as for cumulative readings with modified numerals).

- The attested cumulative reading immediately falls out from the polarity-sensitive definition of the cumulativity operator, repeated below:

(90) $$\begin{aligned} {}^{**}R(X,Y) \iff\ & \forall x \in At^{+}(X), \exists y \in At^{+}(Y), R(x,y) \\ & \wedge \forall y \in At^{+}(Y), \exists x \in At^{+}(X), R(x,y) \\ & \wedge \neg\exists x \in At^{-}(X), y \in At^{\pm}(Y), R(x,y) \\ & \wedge \neg\exists y \in At^{-}(Y), x \in At^{\pm}(X), R(x,y) \end{aligned}$$

- For students $s_1, s_2, s_3$ (where $s_3 =$ Stan ), books $b_1, b_2, b_3$ (where $b_3 =$ Syntactic Structures).

(91) All the students but Stan read all the books but Syntactic Structures.

$$\exists X \in [\![\text{all students but Stan}]\!], \exists Y \in [\![\text{all books but SS}]\!],^{**} R(X,Y)$$
$$\Rightarrow \exists X \in \{s_1^+ \sqcup s_2^+ \sqcup s_3^-\}, Y \in \{b_1^+ \sqcup b_2^+ \sqcup b_3^-\},^{**} R(X,Y)$$
$$\Rightarrow^{**} R(s_1^+ \sqcup s_2^+ \sqcup s_3^-, b_1^+ \sqcup b_2^+ \sqcup b_3^-)$$
$$\Rightarrow \overbrace{\forall s \in \{s_1, s_2\}, \exists b \in \{b_1, b_2\}, R(s,b)}^{\text{otherness+cumulativity}}$$
$$\wedge \forall b \in \{b_1, b_2\}, \exists s \in \{s_1, s_2\}, R(s,b)$$
$$\wedge \neg\exists b \in \{b_1, b_2, b_2\}, R(s_3, b)$$
$$\underbrace{\wedge \neg\exists s \in \{s_1, s_2, s_3\}, R(s, b_3)}_{\text{negative entailments}}$$

- Compositionally, exceptives are defined in terms of a *selective* I-level negation $\sim$.

(92) Given an individual $x \in D$:

$$\sim_x (y^+) = \text{if } y = x \text{ then } y^- \text{ else } y^+$$
$$\sim_x (y^-) = \text{if } y = x \text{ then } y^+ \text{ else } y^-$$

- *but*-exceptives cycle through a group of polarized individuals, and flip the polarity of any atom matching the excepted individual.

(93) $[\![\text{but } x]\!] = \lambda Y \in D^{\pm} . \bigsqcup \{ \sim_x (y) \mid y \leq_A tY \}$

- $[\![\text{but } a]\!] \, (a^+ \sqcup b^+) = a^- \sqcup b^+$
- $[\![\text{but } a]\!] \, (a^- \sqcup b^+) = a^+ \sqcup b^+$
- $[\![\text{but } a]\!] \, (b^+ \sqcup c^-) = b^+ \sqcup c^-$

- Compositionally, one way for composition to proceed is to allow the exceptive to apply to the trace of the moved quantifier:

(94) Every boy but Stan sneezed.
$\exists X \in [\![\text{every boy}]\!], \Delta([\![\text{sneezed}]\!])([\![\text{but Stan}]\!](X))$

- For universal and negative quantifiers, this will immediately deliver the desired results.
- There is independent evidence in favour of a DP-level analysis (due to Moltmann 1995).

(95) Every boy's mother but Annie arrived on time.

# Loose ends

- Still on the docket:
    - An account of the restricted distribution of *but*-exceptives.
    - Exceptives and plural predication.
    - 'All but exactly two….'
- Much work still to be done here!!

# Conclusion

## Summing up

- I've proposed that enriching the domain of individuals with a notion of *polarity* solves a range of problems in the semantics of numeral expressions.
- This picture can be more radically extended to quantificational determiners more generally.
- Treating determiners as *predicates* of groups of polarized individuals constitutes a bona fide alternative to Generalized Quantifier theory, the prevailing approach to the semantics of DPs since the 70s.
- Many consequences and applications have yet to be explored - this is the only the beginning!

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Any mistakes are my own.

*Fin*

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